Engineering Notes

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Impact of Torpedoes on Nets

Andrzej Wortman*
WARS Inc., Santa Monica, Calif.

Nomenclature

C_D = drag coefficient, taken here as unity
d = diameter of the strand
N = effectiveness of interconnection forces, i.e., number of neighboring strands participating in the motion of the strand in question
q_∞ = dynamic pressure $=\rho_\infty U_\infty^2/2$
R_N = nose radius of the torpedo
R_0 = hydrodynamic resistance $=\rho_\infty C_D dN/2$
t = time
T = tension
T_0 = initial tension
U_∞ = speed of the impacting projectile
x = longitudinal coordinate
z = transverse coordinate
θ = dimensionless tension, defined in Eq. (19)
λ = penetration depth $=U_\infty t/R_N$
ξ = dimensionless variable defined in Eq. (7)
ρ = density of the medium
ρ^* $=\rho_T \pi d^2/4$, mass per unit length
ρ_T = density of the string, including virtual mass

Introduction

THE problem of the impact of an object arises in situations requiring either the knowledge of the arrival of an underwater projectile or the necessity of stopping it. In both cases, arrays of strands are placed in the path of the projectile. Distribution of strand tension forces in time and space is necessary for the design of the net itself and the location of sensors to detect the arrival of the projectile.

Analysis

The initial impact of a torpedo on a net results in tension waves which are propagated outward to the boundaries. Individual strands of the net behave like elements of a string under tension. Resistance to motion is the sum of the inertial and hydrodynamic components. Interconnections among the strands act as additional loading which is accounted for by allocating an effective number N of strand elements which participate in the dynamics of the impact phenomena.

Equations of motion of a string with hydrodynamic resistance are given in Morse and Feshbach[1] as:

$$\rho^* \frac{\partial^2 z}{\partial t^2} + R_0 \left(\frac{\partial z}{\partial t}\right)^2 = \frac{\partial}{\partial x}\left(T\frac{\partial z}{\partial x}\right) \tag{1}$$

It is shown in Timoshenko[2] that the stress in a rod struck transversely by a force moving a speed v is proportional to v.

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*President.

Thus the solution sought is such that

$$T/R_0 = k\frac{\partial z}{\partial t} \tag{2}$$

with k being determined from the solution. Near the point of impact, the transverse motion and the viscous forces dominate the inertial contributions which are, therefore, neglected. The region of validity of this approximation will be estimated later.

The equation of motion is now written as

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial}{\partial z}\ell n T\left(\frac{\partial z}{\partial z}\right)^2 = \frac{1}{k}\frac{\partial t}{\partial z} \tag{3}$$

with the transformation[3]

$$\theta = \int_0^z \frac{T}{T_0}\mathrm{d}z = \int_0^z \tau \mathrm{d}z \tag{4}$$

the equation is reduced to

$$\theta_{xx} = \frac{1}{k}\theta_{tt} \tag{5}$$

where the subscripts denote partial derivatives.

This is the diffusion equation commonly encountered in transient heat conduction. Numerous classical solutions exist with the most appropriate one being that for a linear change in temperature, which corresponds exactly to the initial impact of the torpedo prior to any significant penetration. For a temperature rise given by At, with A constant[1].

$$\theta = At\left[\left(1+2\xi^2\right)\mathrm{erfc}\xi - \frac{2}{\sqrt{\pi}}\xi e^{-\xi^2}\right] \tag{6}$$

with

$$\xi = x/(2\sqrt{kt}) \tag{7}$$

the condition

$$x=0 \qquad \theta = At$$

yields

$$A = k\frac{R_0}{T_0}U_\infty^2 \tag{8}$$

which now relates the transient temperature analog to the torpedo speed U_∞. It should be noted that the force balance

$$\int_{x_0}^\infty R_0\left(\frac{\partial z}{\partial t}\right)^2 \mathrm{d}x = -T\frac{\partial z}{\partial x}\Big|_{x_0} \tag{9}$$

is satisfied exactly. Therefore, the analogy is exact and the results may be used directly. Transverse velocity of the string is now found to be

$$\left(\frac{\partial z}{\partial t}\right)^2 = U_\infty^2 \mathrm{erfc}\xi \tag{10}$$

The remaining task is the evaluation of the constant k in Eq. (2). This is done by equating the energy expended by the strand moving at the point of impact with the energy given up by the torpedo. The normal force at the center

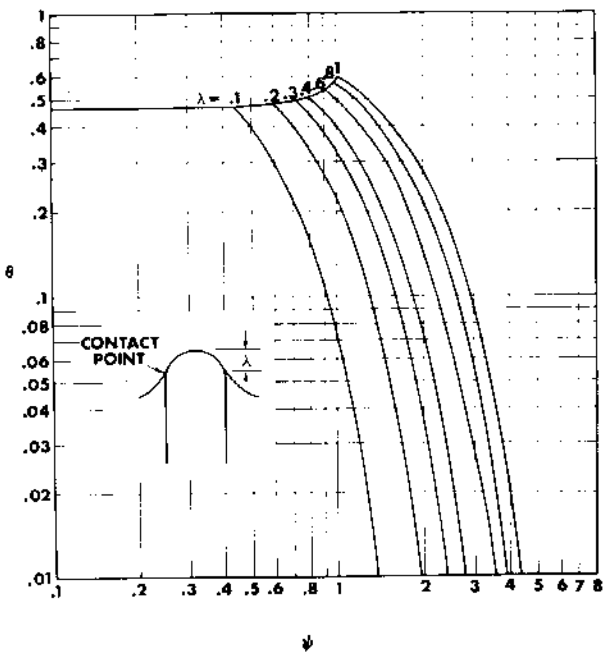

Fig. 1 Distribution of the tension variable with radial extent at fixed values of penetration parameter.

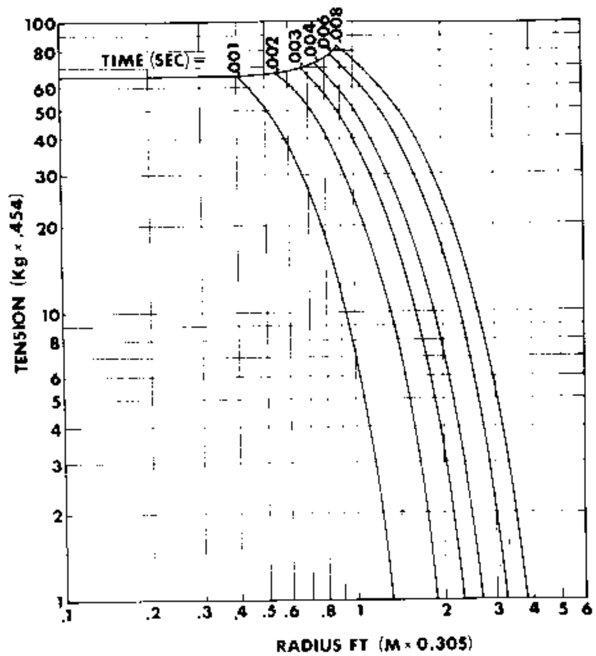

Fig. 2 Distribution of tension with radius at various times after impact.

$$F_N = -T\cdot\frac{\partial z}{\partial x}\bigg|_{x_0} \tag{11}$$

moves at the speed U_∞, so that in time t the work done is

$$W_1 = \frac{4}{3\sqrt{\pi}}\sqrt{k}R_0U_\infty^{3}t^{3/2} \tag{12}$$

at the point of contact of the strand with a spherical nose of the torpedo the slope is

$$\left(\frac{dy}{dx}\right) = (2\lambda)^{-1/2} \tag{13}$$

the normal force

$$F = T\left(\frac{dy}{dx}\right)^{-1} \tag{14}$$

and the work done at time t is

$$W_2 = \frac{2\sqrt{2}}{3}\frac{kR_0U_\infty^{3/2}}{\sqrt{R_N}}t^{3/2} \tag{15}$$

From the equality of the energy expenditures

$$k = 2R_NU_\infty/\pi \tag{16}$$

the tension T in the strand is, therefore, given by

$$T = \frac{2qR_N^2C_dN}{\pi}\left(\frac{d}{R_N}\right)\left[\operatorname{erfc}\left(\sqrt{\frac{\pi}{8}\frac{\psi}{\sqrt{\lambda}}}\right)\right]^{1/2} \tag{17}$$

The tension is highest at the contact point on the projectile surface which is calculated to be:

$$\psi_c = (2\lambda-\lambda^2)^{1/2} \tag{18}$$

The dimensionless tension

$$\theta = \pi T/(2qdR_NC_DN) \tag{19}$$

Can be expressed as a function of ψ with λ as a parameter. This is shown in Fig. 1. In order to establish magnitudes of the quantities involved, specific cases were calculated for the following arbitrarily selected variables

$R_N = 26.7$ cm (10.5 in.)	$U_\infty = 30.5$ m/s (100 ft/s)
$d = 0.127$ cm (0.05 in.)	$\rho_\infty = 1$ g/cm^3 (2 slug/ft^3)
$N = 5$	

The results are shown in Fig. 2. It is now necessary to establish the range of validity of the derived solutions by comparing the relative magnitudes of inertial and dissipative forces:

$$E = \frac{\text{inertia}}{\text{viscous forces}} = \frac{\rho^* z_{tt}}{R_0(z_t)^2} = \left(\frac{\sqrt{\pi}}{2}\frac{\rho_T}{\rho_\infty}\frac{l}{C_dN}\frac{d}{R_N}\right)\frac{l}{\lambda}\frac{\xi e^{-\xi^2}}{(\operatorname{erfc}\xi)^{3/2}} \tag{20}$$

When the value of E approaches unity, the results obtained here will become inaccurate. Solution of Eq. (20) for the physical variables shown above indicates that the dimension radius ψ at which E reaches the value of unity is about seven times the value of the penetration variable λ for all values of λ of interest.

The solution presented here approaches the problem of impact of an underwater projectile on a net under the assumed limiting conditions of a sparse array of strands so that, essentially, only one strand is involved and the influence of its neighbors appears weakly only through the factor N. The opposite limit of a very dense array is also of some practical interest and can be attacked using the technique illustrated here.

References

[1] Morse, P.M. and Feshbach, H., *Methods of Theoretical Physics*, Vols. I and II, McGraw-Hill Book Co., N.Y., 1953.

[2] Timoshenko, S., *Strength of Materials*, Vol. 1, R. Kreiger, N.Y., 1976, p. 320.

[3] Carslaw, H.S., and Jaeger, J.C., *Conduction of Heat in Solids*, 2nd ed., Oxford Press, Oxford, England, 1959, p. 89.